POST-GRADUATE DIPLOMA IN STATISTICAL METHODS AND ANALYTICS

TEST CODE: DST (MCQ) 2016

SYLLABUS

Algebra—Arithmetic, Geometric and Power series, sequences. Permutations and combinations. Binomial theorem. Theory of quadratic equations. Inequalities. Elementary set theory. Vectors and matrices. Determinant, rank and inverse of a matrix. Solutions of linear equations.

Coordinate geometry — Straight lines, circles, parabolas, ellipses and hyperbolas.

Calculus—Taylor and Maclaurin series. Limits and continuity of functions of one real variable. Differentiation and integration of functions of one real variable with applications. Definite integrals. Areas using integrals. Maxima and minima and their applications.

SAMPLE QUESTIONS

Note: *For each question there are four suggested answers of which only one is correct.*

1. If a focal chord of the parabola $y^2 = 4ax$ cuts it at two distinct points (x_1, y_1) and (x_2, y_2), then

   (A) $x_1x_2 = a^2$ (B) $y_1y_2 = a^2$ (C) $x_1x_2^2 = a^2$ (D) $x_1^2x_2 = a^2$.

2. Consider a circle with centre at origin and radius $2\sqrt{2}$. A square is inscribed in the circle whose sides are parallel to the X and Y axes. The coordinates of one of the vertices of this square are

   (A) $(2, -2)$ (B) $(2\sqrt{2}, -2)$ (C) $(-2, 2\sqrt{2})$ (D) $(2\sqrt{2}, -2\sqrt{2})$.

3. Let $a_n = \left(1 - \frac{1}{\sqrt{2}}\right) \cdots \left(1 - \frac{1}{\sqrt{n+1}}\right)$, $n \geq 1$. Then $\lim_{n\to\infty} a_n$

   (A) equals 1 (B) does not exist (C) equals $\frac{1}{\sqrt{\pi}}$ (D) equals 0.

collegedunia
India's largest Student Review Platform

4. Let $S = \{1, 2, \ldots, 100\}$. Two integers are chosen from S such that their sum is odd. The number of ways these integers can be chosen is
(A) 1250 (B) 2500 (C) 2450 (D) 1225.

5. The integral
$$\int_0^{\frac{\pi}{2}} \frac{\sin^{50} x}{\sin^{50} x + \cos^{50} x}\, dx$$
equals

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) none of these.

6. Let x, y be non-zero real numbers such that $x^2 > y^2$. Then which of the following inequalities is true?
(A) $x > y$ (B)$-x < -y$ (C) $|x| > |y|$ (D) $-|x| > -|y|$.

7. If the sum of first n terms of an arithmetic progression is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$
(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$.

8. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is equal to

(A) 1 (B) 2 (C) 3 (D) none of these.

9. The set $\{x \in \mathbb{R} : x^2 > |x|\}$ is
(A) $(1, \infty)$ (B) $\mathbb{R} \backslash (0, 1)$ (C) $\mathbb{R} \backslash [-1, 1]$ (D) $\mathbb{R} \backslash (-1, 1)$.

10. The number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000 is

(A) 40 (B) 50 (C) 60 (D) 30.

11. Let the function $f(x)$ be defined as $f(x) = |x-1| + |x-2|$. Then which of the following statements is true?
(A) $f(x)$ is differentiable at $x = 1$

collegedunia
India's largest Student Review Platform

(B) $f(x)$ is differentiable at $x = 2$

(C) $f(x)$ is differentiable at $x = 1$ but not at $x = 2$

(D) none of the above.

12. $x^4 - 3x^2 + 2x^2y^2 - 3y^2 + y^4 + 2 = 0$ represents

(A) A pair of circles having the same radius

(B) A circle and an ellipse

(C) A pair of circles having different radii

(D) none of the above.

13. Consider the following two series $a, a+b, a+2b, \ldots$ and $b, b+a, b+2a, \ldots$, where a and b are real numbers with $a \neq b$. Let S_1 and S_2 be the sum of the first n terms for these two series, respectively. Then $S_1 = S_2$ for

(A) $n = 2$ (B) $n = 3$ (C) all values of n (D) no value of n.

14. The equation $ax^2+bx+c = 0$ where $a \neq 0$, has two real roots which are equal in magnitude but opposite in sign. Then which of the following is true?

(A) $b = 0, c \neq 0$ (B) $b = c = 0$ (C) $b \neq 0, c \neq 0$ (D) $b \neq 0, c = 0$.

15. The sum of the series $\dfrac{1}{1.2} + \dfrac{1}{2.3} + \ldots + \dfrac{1}{n(n+1)} + \ldots$ is

(A) 1 (B) 1/2 (C) 0 (D) non-existent.

16. Let $F(x) = \displaystyle\int_0^x \frac{t^2 dt}{(1+t^3)^2}$. Then the value of $\displaystyle\lim_{x\to\infty} F(x)$ is

(A) 1/3 (B) $-1/3$ (C) 0 (D) 2/3.

17. $A = (4, 1)$, $B = (2k+1, 3k)$ and $C = (k+1, 2k)$ are 3 distinct points in the X-Y plane. These three points lie in a straight line if the value of k is

(A) 0 (B) -2 (C) 1/2 (D) $-1/2$.

18. Let $f(x) = \dfrac{2x}{x-1}$, $x \neq 1$. State which of the following statements is true.

(A) For all real y, there exists x such that $f(x) = y$;

(B) For all real $y \neq 1$, there exists x such that $f(x) = y$;

collegedunia India's largest Student Review Platform

(C) For all real $y \neq 2$, there exists x such that $f(x) = y$;

(D) None of the above is true.

19. The determinant $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$ equals

(A) $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(B) $2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(C) $3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

(D) None of these.

20. Let $f(x) = 17x^7 - 19x^5 - 1$, where x is real. Then
(A) $f(x)$ has no real root
(B) $f(x)$ has a real root in the interval $-1 < x < 0$
(C) $f(x)$ has no positive real root less than 2
(D) $f(x)$ can have only negative real roots.

21. Let $A = \{x \in \mathbb{R} : |x - 2| < 1\}$ and $B = \{x \in \mathbb{R} : x^2 - 3x + 2 > 0\}$. Then $A \cap B$ is the interval
(A) $[1, 3]$ (B) $[2, 3]$ (C) $(1, 3)$ (D) $(2, 3)$.

22. The area enclosed by the curve $|x| + |y| = 1$ is

(A) 1 (B) 2 (C) $\sqrt{2}$ (D) 4.

23. Let $(m_i, \frac{1}{m_i}), m_i \neq 0, i = 1, 2,$ be two distinct points on the straight line $ax + by + c = 0$. Then the value of $m_1 m_2$ is
(A) c/a (B) $-c/a$ (C) b/a (D) $-b/a$.

24. A pizza company gives free home delivery within a 5 km radius of its shop. Suppose in a map, with a coordinate system having 1 km as unit, the shop is located at coordinates $(2, 3)$. The coordinates of 4 homes on the same map are $(1, 5)$, $(4, 6)$, $(-2, -5)$ and $(3, -6)$. Then the number of homes qualifying for free home delivery is
(A) 1 (B) 2 (C) 3 (D) 4.

collegedunia
India's largest Student Review Platform

25. The number of real roots of the equation $1+\cos^2 x+\cos^3 x-\cos^4 x=5$ is

(A) 0 (B) 1 (C) 3 (D) 4.

26. Let $\Omega=\{a,b,c,d\}$ and $A\subseteq\Omega$. The number of such subsets A such that the set $(A\cup\{a,b\})\backslash(A\cap\{a,b\})$ has exactly one element is

(A) 1 (B) 2 (C) 3 (D) 4.

27. Consider the following system of equations in 3 variables x,y,z.

$2x+2y+6z=8$

$x+2y+\lambda z=5$

$x+y+3z=4$

where λ is a real constant. Then the system has

(A) no solution.

(B) a unique solution for all values of λ

(C) a unique solution for a particular value of λ

(D) infinite number of solutions.

28. ${}^nC_0+2{}^nC_1+3{}^nC_2+\ldots+(n+1){}^nC_n$ equals

(A) 2^n+n2^{n-1} (B) 2^n-n2^{n-1} (C) 2^n (D) none of these.

29. The sum of the lengths of the three sides of a $\triangle ABC$ is 5 units. Suppose $\sin\angle BAC=a/5$, where a is the length of the side BC. Then

(A) $\sin\angle BAC+\sin\angle ABC+\sin\angle BCA=1$

(B) $\sin\angle BAC+\sin\angle ABC+\sin\angle BCA>1$

(C) $\tan\angle BAC+\tan\angle ABC+\tan\angle BCA>1$

(D) $\tan\angle BAC+\tan\angle ABC+\tan\angle BCA=1$.

30. Let $f(x)=\min_{x\in[0,2]}\{x,x^2\}$. Then $\int_0^2 f(t)dt$ is equal to

(A) 5/6 (B) 11/6 (C) 8/3 (D) 2/3.

31. Suppose $f(n+1)=f(n)+n^2,\ n=0,1,2,\ldots$ and $f(0)=1$. Then the value of $f(11)$ is

(A) 386 (B) 286 (C) 506 (D) 385.

32. Consider an experiment in which two dice are simultaneously thrown. We say a *Double* appears if both the dice show the same face. This

collegedunia
India's largest Student Review Platform

experiment is repeated three times. The number of times when exactly 2 *Doubles* can appear in these 3 repetitions is

(A) 18 (B) 12 (C) 3240 (D) 108.

33. The area bounded by the curves $y = x^2$ and $y = 8 - x^2$ is equal to

(A) 64 (B) 128/3 (C) 64/3 (D) 32.

34. A number is a palindrome if it is the same when read from left to right or when read from right to left. For example, 242 is a 3-digit palindrome and 10001 is a 5-digit palindrome. Then the total number of 7-digit palindromes which can be formed by using the digits 0,1,...,9 is

(A) 10^4 (B) 10^3 (C) 9×10^2 (D) 9×10^3.

35. The value of $\int_{-1}^{1} x|x|e^{x^2}dx$ is equal to

(A) e^{-1} (B) $2e$ (C) 0 (D) e.

36. Consider all possible arrangements of all the letters of the word **CRICKET**. If these arrangements are now listed in the alphabetical order, then the position of the word **CRICKET** in this list is

(A) 2520 (B) 531 (C) 771 (D) 530.

37. The coefficient of x^3y^2 in the expansion of $(5 + 2x + 3y^2)^5$ is

(A) 20 (B) 480 (C) 800 (D) 2400.

collegedunia
India's largest Student Review Platform

38. Consider all possible 7-digit numbers formed by using digits from the set$\{1, 2, 3\}$ such that the sum of the seven digits is equal to 10. Then the total number of such 7-digit numbers is
(A) 3^7 (B) 77 (C) 7^3 (D) 33.

39. Let $f(x) = \sin x^2$, $x \in \mathbb{R}$. Then
(A) f has no local minima;
(B) f has no local maxima;
(C) f has local minima at $x = 0$ and $x = \pm\sqrt{(k + \frac{1}{2})\pi}$ for odd integers k and local maxima at $x = \pm\sqrt{(k + \frac{1}{2})\pi}$ for even integers k;
(D) None of the above.

40. Let $f(x) = \sqrt{2 - x} + \sqrt{1 + x}$ be a real valued function. Then the range of $f(x)$ is the interval
(A) $[\sqrt{3}, \sqrt{6}]$ (B) $(\sqrt{2}, \sqrt{3})$ (C) $(\sqrt{3}, \sqrt{6})$ (D) $(3, 6)$.

41. The equation $2x^2 + 2y^2 + 5xy + 3x + 3y + 1 = 0$ represents
(A) a circle (B) an ellipse (C) a parabola (D) a pair of straight lines.

42. The set $S = \{(x, y) \in \mathbb{R} \times \mathbb{R}, x^2 + y^2 = 1$ and $x,\ y$ are both rational$\}$
(A) is the empty set.
(B) has exactly two points.
(C) has exactly four points.
(D) has infinitely many points.

43. Let $n > 1$ be a positive integer. Then
(A) $n(n + 1)^2 \geq 4(n!)^{3/n}$
(B) $n^2(n + 1)^2 < 4(n!)^{3/n}$
(C) $n(n + 1) > 4(n!)^{3/n}$
(D) $(n + 1)^2 \geq 4(n!)^{3/n}$.

44. Suppose the circle with equation $x^2 + y^2 + 2fx + 2gy + c = 0$ cuts the parabola $y^2 = 4ax$, $(a > 0)$ at four distinct points. If d denotes the sum of the ordinates of these four points, then the set of possible values of d is
(A) $\{0\}$ (B) $(-4a, 4a)$ (C) $(-a, a)$ (D) $(-\infty, \infty)$.

collegedunia
India's largest Student Review Platform

45. Consider the geometric figure enclosed by the bisectors of the four angles of any parallelogram. Then the *most appropriate* description of this figure is

(A) parallelogram (B) rectangle (C) rhombus (D) quadrilateral.

46. Consider all possible words obtained by arranging all the letters of the word **AGAIN**. These words are now arranged in the alphabetical order, as in a dictionary. The fiftieth word in this arrangement is

(A) IAANG (B) NAAGI (C) NAAIG (D) IAAGN.

47. Let $y = [\log_{10} 3245.7]$ where $[a]$ denotes the greatest integer less than or equal to a. Then

(A) $y = 0$ (B) $y = 1$ (C) $y = 2$ (D) $y = 3$.

48. The number of integer solutions for the equation $x^2 + y^2 = 2011$ is

(A) 0 (B) 1 (C) 2 (D) 3.

49. The number of ways in which the number 1440 can be expressed as a product of two factors is equal to

(A) 18 (B) 720 (C) 360 (D) 36.

50. For the matrices $A = \begin{pmatrix} a & a \\ 0 & a \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $(B^{-1}AB)^3$ is equal to

(A) $\begin{pmatrix} a^3 & a^3 \\ 0 & a^3 \end{pmatrix}$ (B) $\begin{pmatrix} a^3 & 3a^3 \\ 0 & a^3 \end{pmatrix}$

(C) $\begin{pmatrix} a^3 & 0 \\ 3a^3 & a^3 \end{pmatrix}$ (D) $\begin{pmatrix} a^3 & 0 \\ -3a^3 & a^3 \end{pmatrix}$

collegedunia
India's largest Student Review Platform